

Integral

Math Figures

A History of Mathematics

Produced by the AP Calculus Class

of

Manhattan High School for Girls

June 2017

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A history of famous Math figures
produced by the AP Calculus Class
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Message from the Principal

German mathematician Gottfried Wilhelm von Leibniz who co-discovered calculus stated, “Music is the pleasure the human soul experiences from counting without being aware that it is counting.” Pure mathematics is the search for highly structured beauty. The history of math, then, is like the history of art, or of poetry. The concepts we take for granted like line, function, and infinity changed over time, and studying math’s history illuminates its beauty.

I am so proud of this year’s math journal because it reflects our girls’ deep understanding and contextualization of the purpose of the math they have studied this year in their AP Calculus course. Thank you to Mrs. Goldie Feinberg, their teacher and our Math Chair, for brilliantly giving our girls the practical applications to conceptual mathematics.

Mrs. Estee Friedman-Stefansky
Principal
General Studies

Foreword

“The definition of a good mathematical problem is the mathematics it generates rather than the problem itself,” said Andrew Wiles, the mathematician who finally discovered a proof to “Fermat’s Last Theorem.” Intelligent, curious people throughout history tried to answer questions, both theoretical and practical, that resulted in many mathematical ideas and formulae. Some solved the problems they set out to fix, others did not, but the process led to many mathematical truths that have impacted the math and science knowledge we have today. Understanding the context of how and when math evolved helps us understand mathematics and appreciate its beauty. Many students see mathematics as an endless sea of ideas that seem independent of each other. Exploring the history of math and how it evolved helps disentangle the jumble of ideas and portrays the flow of math and how each newly discovered idea contributed to the next area of development. My hope is that my students will always use their innate gifts, curiosity, and acquired skills in a way that will not only answer the intellectual challenges they will encounter but that will pose new challenges and opportunities for themselves and the world at large.

Mrs. Goldie Feinberg
Chair, Math Department

Pythagorus

570

Euclid

325

Archimedes

287

Descartes

1596

Fermat

1601

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Newton
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1643

Leibniz
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1646

Euler
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1707

Gauss
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1777

Reimann
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1826

570 BCE - 495 BCE

Pythagoras

Chashie Komendant

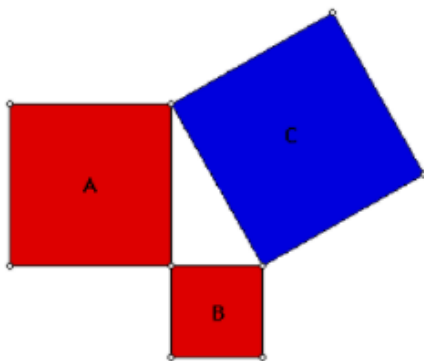
Thousands of students a day use the famous Pythagorean Theorem, $a^2 + b^2 = c^2$. However, more than ninety percent of the students do not know how it came about. Pythagoras was born in Greece in the year 570 BCE. on Samos Island and died in 495 BCE.¹ In his youth, he was taught mathematics by



Thales and was a disciple of Anaximander, another student of Thales. When he was twenty-two years old, he visited Egypt and then lived there for the next twenty two years of his life. After a Persian invasion of Egypt, he was taken prisoner and brought to Babylon and lived there for the next twelve years. During his time in Babylon, he learned mathematics and Eastern spiritual ideas. When he returned to Samos at the age of fifty-six, he taught people philosophy of life, “based on a mixture of his own ideas, mathematics and mysticism from Ancient Egypt and the East.” After two years in Samos, he left to Croton because no one was open to his teachings. There, he established the Pythagoreans, a “religious sect or cult whose beliefs were based on the power of numbers; honesty; living a simple, unselfish life; and generally trying to show kindness to people and animals.”² The brotherhood of Pythagoreans was devoted to the study of mathematics, and no one is able to tell if Pythagoras is the actual one who came up with the theorem.³ The theorem was a mathematical fact that the Babylonians knew, however Pythagoras was the first to prove the theorem and therefore was given credit for the first proof.⁴

The theorem itself is the most famous mathematical contribution.

The Pythagorean theorem says, “The area of the square built upon the hypotenuse of a right triangle is equal to the sum of the areas of the squares upon the remaining sides.” In the figure on the left, the sum of the areas of the two right red squares, squares A and B, is equal to the area of the blue square, square C. Even



though Pythagoras was credited with the theorem, it was likely that the Babylonians knew the result for certain specific triangles earlier than Pythagoras. The Pythagorean Problem is to find all the right triangles whose sides are of integral length, thus finding all solutions in the positive integers of the Pythagorean equation $x^2 + y^2 = z^2$.⁵ When the brotherhood of Pythagoreans learned that this creates irrational numbers, for example the shorter lengths of the triangle are both equal to 1, the hypotenuse is $\sqrt{2}$. This was a shock to the brotherhood, and they tried to keep it quiet, legend even has it that they killed anyone outside of their group if they learned that irrational numbers exist.⁶ So we students are not alone in our struggle in trying to rationalize irrational numbers. The struggle began centuries ago with the discovery of $\sqrt{2}$.

1 <http://www.iep.utm.edu/pythagor/>

2 <https://www.famousscientists.org/pythagoras/>

3 <http://www.geom.uiuc.edu/~demo5337/Group3/hist.html>

4 <http://www.ms.uky.edu/~lee/ma502/pythag/pythag.htm>

5 http://jwilson.coe.uga.edu/emt669/student_folders/morris.stephanie/emt.669/essay.1/pythagorean.html

6 <https://www.famousscientists.org/pythagoras/>

325 BCE - 265 BCE

Euclid of Alexandria

Elisheva Rosensweig

Euclid was born in Egypt in c. 300 BCE and is known to have taught in Alexandria under the rule of Ptolemy I Soter.

He is most famous for his written work, *Elements*, discussing Geometry. It was through this book that he then came to be known as “The Father of Geometry.”

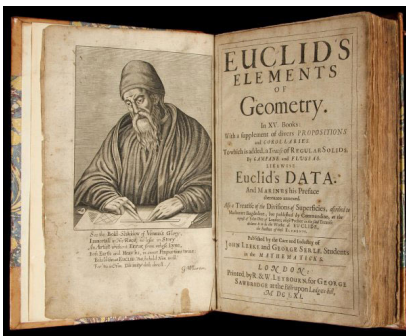
Upon discussing his works with Ptolemy, “Ptolemy once asked Euclid if there was not a shorter road to geometry than through the *Elements*, and Euclid replied that there was no royal road to geometry.”

Although Euclid was definitely influenced by his predecessors, he is known to have made his discoveries independently. He also believed that his mathematical discoveries were reflections of Hashem’s work and that “the laws of nature are but the mathematical thought of G-d”

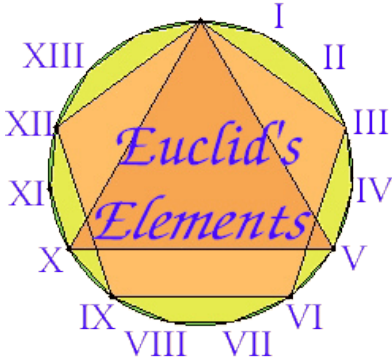
Euclid’s *Elements* consists of 13 volumes which discuss the construction of five regular solids, known as platonic solids. It is the second oldest existing greek mathematical text. A common misconception is that *Elements* only discusses concepts of geometry (which is true about

book one through four), however, the postulates, theorems, and concepts also relate to algebra.¹

Elements is known to be one of the most influential works on mathematics until the late 19th or early 20th century. During this time period it served as the main mathematics textbook.²



The first volume of *Elements* begins by defining many geometry concepts such as a line or a point. These definitions are used throughout every geometry class, and provided a standard reference point for mathematicians, to have the same definitions of these known concepts. Euclid also discusses five unproved assumptions known as postulates and five unproved assumptions he called common notions. These include the concept that “given any two points such as A and B, there is a line AB which has them as endpoints,”³ defining right angles, declaring they are all equal, and the famous Transitive theory (If $a = b$ and $b = c$, then $a = c$).⁴ These are the foundations in which Geometry is built upon. In later volumes, Euclid proves theorems of parallelograms and triangles like the pythagorean theorem. He also discusses geometric algebra, circles, construction of regular polygons, ratios and proportions, and three dimensional figures.



Euclid’s approach to writing and proving his discoveries in *Elements*, was done in a series of statements that then built upon one another to create proofs and postulates. He would begin with definitions of points, planes, and lines. Using his original definition, he would create and prove more definitions while always returning to the original definition used in the beginning. Through using this method, Euclid created a clear set of mathematical proofs and definitions, all clearly proven step by step from the original known facts. Using this complex method, he goes on to prove and create platonic solids, which are extremely

complicated, but through using only previously proven concepts and ideas in his *Elements*. The method in which we learn Geometry in MHS is based on this approach.

1 <https://www.britannica.com/biography/Euclid-Greek-mathematician>

2 Hoffman, Mike. "Bernhard Riemann" U. S. Naval Academy. 6 June 2017. Web. <https://www.usna.edu/Users/math/meh/riemann.html> Taisbak, Christian Marinus, and Bartel Leendert Van Der Waerden. "Euclid." Encyclopædia Britannica. Encyclopædia Britannica, inc., 25 Jan. 2017. Web. 24 May 2017.

3 <http://www.thefamouspeople.com/profiles/euclid-436.php> "Who is Euclid? Everything You Need to Know." Childhood, Life Achievements & Timeline. N.p., n.d. Web. 24 May 2017.

4 "Postulate 1." Euclid's Elements, Book I, Postulate 1. N.p., n.d. Web. 24 May 2017.

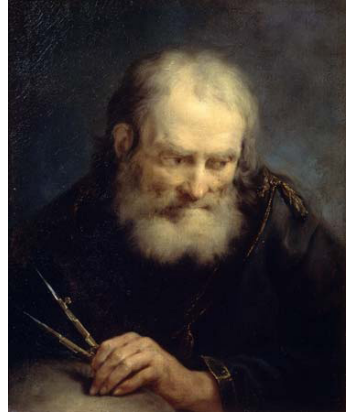
5 www.freemathhelp.com/forum/threads/93383-If-a-b-and-b-c-then

287 BCE. - 212 BCE

Archimedes

Serene Klapper

Archimedes was born in 287 BCE in Syracuse.¹ From a young age Archimedes was constantly building and calculating. He researched many mathematical principles in order to make discoveries that would help him apply these concepts to life. Most of Archimedes' inventions, such as the hydraulic screw and weapons for his country, were based on mathematical concepts that he studied.³



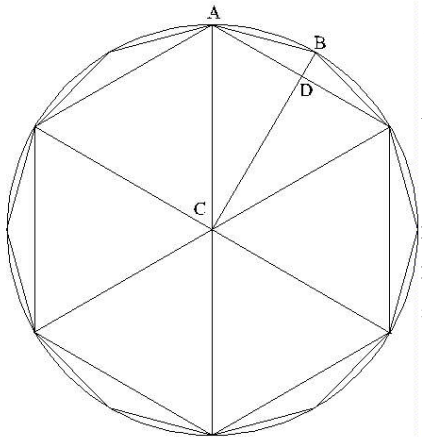
As a young man, Archimedes traveled to Alexandria, Egypt for his studies. There he studied the works of Euclid, whose writing influenced many of Archimedes's own. When he returned to Syracuse, Archimedes worked as the King's "problem solver," applying his mathematical knowledge, and inventive mind toward resolving the King's crises.²

Although Archimedes invested a lot of time and effort into his practical inventions, most of his writings focused on pure mathematical concepts. In one of his more famous works, he approximated the value of pi more accurately than ever before.¹ This estimation made it possible for mathematicians to calculate volumes and areas of solids and shapes with curved lines, which later lead to the invention of integral calculus. In fact, Archimedes is considered "the father of integral calculus."⁶

Archimedes was able to obtain this very accurate approximation using a theorem from Euclid's "Elements." Based off this work, Archimedes wrote a formula enabling the calculation of a polygon with $2n$ sides, when the perimeter of a polygon with n sides is given. Beginning with a circumscribing polygon with $n=6$ sides, he calculated

the perimeter of a polygon with first 12, then 24, 48, and 96 sides. Through experimenting with angle bisectors and Euclid’s theorem, Archimedes was able to deduce that, “*The ratio of the circumference of any circle to its diameter is less than $3^{1/7}$ but greater than $3^{10/71}$.*” Although he wrote this in his treatise “Measurement of A Circle,” Archimedes skipped many steps in his writings, forcing mathematicians to research how he received his results. However, that was simply part of Archimedes competitive personality. He enjoyed challenging others by giving them the answer , and forcing them to backtrack to discover how to get that solution.⁶

Archimedes was in the midst of a calculation when his Sicily was invaded by Roman soldiers either 211 BCE or 212 BCE. The emperor had heard of his brilliance, and demanded Archimedes be captured and killed. Archimedes was in the midst of a calculation when the Roman soldiers apprehended him, and insisted he accompany them. Unwilling to abandon his calculations, Archimedes refused, and was murdered by the dissatisfied soldier.²



5

1. <https://www.britannica.com/biography/Archimedes>
 2 <http://archimedespalimpsest.org/about/history/archimedes.php>
 3 <https://www.cs.drexel.edu/~corres/Archimedes/contents.html>
 4 <http://www.thefamouspeople.com/profiles/archimedes-422.php>
 5 http://galileo.phys.virginia.edu/classes/109/lectures/greek_math.htm
 6 <http://itech.fgcu.edu/faculty/clindsey/mhf4404/archimedes/archimedes.html>

1596 - 1650

Rene Descartes

Estee Gerber

Rene Descartes was born on March 31st, 1596, in La Haye, France to a bourgeois couple. He became known throughout history as the “father of modern philosophy.” His basic premise, “I think therefore I am,” revolutionized the basic understanding of philosophy, and later influenced his mathematical works.



Descartes was not born to an easy life. His mother passed away when he was young, and he and his siblings were sent away to live with their grandmother.

And still the troubles continued. He was diagnosed with a feeble constitution and later on, was rumored to have suffered a nervous breakdown. In 1618, Descartes travelled to the Netherlands to serve in the army of Maurice of Nassau, and there he met Isaac Beekman, who introduced Descartes to the application of mathematics to all studies.

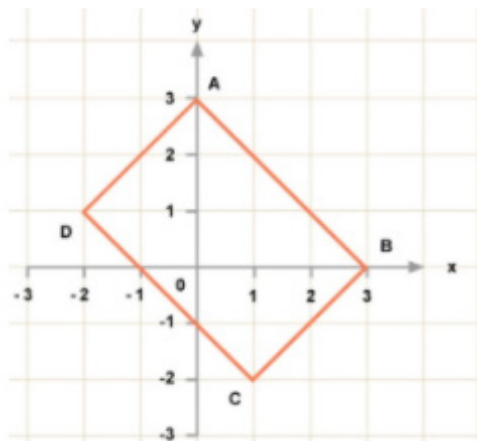
And still the troubles continued. In 1629, Descartes started his work, *The World*, which revealed mechanistic philosophy and that constant rules of nature explain the physical world, and not observances or a dialectic of the subject. Unfortunately, he also wrote about the validity of a heliocentric universe, which contradicted Church geocentric teachings. Since Galileo had recently been persecuted by the Church for similar teachings, Descartes decided not publish this work.

There’s a reason Descartes work is still taught today. He was one of the first to explain events rationally. He built a new foundation for knowledge based on reason, rather than discourse. Descartes’s *Discourse On Method*, was published in 1637 as a preface to three other essays. In

it, he examined the root of our existence and surmised that the only principle that has no doubt must be that because I think, I must exist or I think therefore I am (in Latin, cogito ergo sum).

Descartes' most famous mathematical contribution was the section *La Géométrie* written in his *Discourse on Method*. He composed geometrical calculus, which is the relationship between geometry and algebra. As Beekman taught him, Descartes used his philosophical technique, of understanding the root, and spread it to mathematics. He wanted to break down the study of geometry to its most basic understanding. And here he discovered that geometry, the study of shapes, is really defined by an equation, which is algebra. Additionally, Descartes was the first to express the unknown (x,y,z) in terms of the known (a,b,c) and the known in terms of the unknown in respect to geometrical shapes.

The fundamental ideas we are taught today in elementary school were only first identified by Descartes in the 1600's. Today called Cartesian coordinates, Descartes was the first to institute a series of points to be generated by using a horizontal x-axis and a vertical y-axis, creating a two dimensional plane that could allow for an algebraic equation to identify a series of points. Now one would be able to describe a curve by the position of its points. Furthermore, since it was possible to solve equations graphically instead of algebraically, later on, Newton and Leibniz were



able to find the intersection of two equations of lines and spur the study of calculus.

Although commonly known for his philosophical writings, Descartes' mathematical discoveries helped advance society's understanding of math. As with philosophy, Descartes searched for the essence of the idea. He dissected geometry into a form of algebra, where an equation describes the shape. He came up with the two dimensional graph with four quadrants that would allow for an algebraic equation to classify an array of points. And he came up with the common notation used today of (a,b,c) and (x,y,z) . It is thanks to Descartes, and other mathematicians like him, that we are able to recognize and explain the natural workings of the physical world.

1 The idea that the universe is governed by natural laws.

2 Description based on ideas instead of a description based on a equation.

3 <http://www.iep.utm.edu/descarte/#H3>

4 <https://plato.stanford.edu/entries/descartes-mathematics/#BooOneDesGeoAna>

5 http://www.storyofmathematics.com/17th_descartes.html

1601-1665

Pierre de Fermat

Chana Steinberg

Pierre de Fermat was a successful lawyer, from a wealthy merchant family, who was fluent in many languages. He was so successful, he was made king's councillorship in the parliament of Toulouse, and was allowed to add the "de" to his name, becoming Pierre de Fermat. But by night, he was a brilliant mathematician, significantly impacting calculus and number theory. His mathematical achievements are astonishing when you consider that it was



a hobby, not a life's calling. He didn't publish his work, and much of his math is known today because of letters he wrote to his friend. Because of this he didn't have space to write proofs, and some mathematicians doubt that he actually had proofs for all of his theorems.

Pierre de Fermat was instrumental in many major fields of math, but he also came up with interesting theorems that have puzzled mathematicians for hundreds of years. And together with Pascal he came up with Probability Theory, a useful field at the time because of gambling.

Any student who takes calculus will recognize his work on tangents, on which Newton based his method of tangents. Pierre came up with a method for finding minimum and maximum on polynomial curves, tangents to curves, and for finding the space of the area under power curves.

His most famous work, Fermat's Last Theorem, states that $a^n + b^n = c^n$ for all n is greater than 2 and abc are greater than 0 has no solution. There are infinitely many solutions for $n = 1$ or 2 (think pythagorean

theorem), but although mathematicians believed the theorem to be true, they couldn't figure out how to prove it. Like many of his theorems, it was written in a letter without a proof because there wasn't enough space. Mathematicians grappled with it for centuries, and it wasn't proven till 1994, by Andrew Wiles; the proof was pages long.

1 http://www.storyofmathematics.com/17th_fermat.html

2 <http://sites.math.rutgers.edu/~cherlin/History/Papers2000/pellegrino.html>

3 <http://www.mscs.dal.ca/~kgardner/History.html>

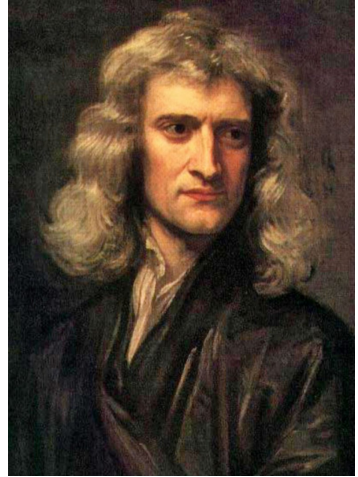
4 <http://famous-mathematicians.org/pierre-de-fermat/>

1643-1727

Sir Isaac Newton

Shalva Gozland

Meet Newton. Born long before you were (year 1643 to be exact) in Lincolnshire, England, Sir Isaac attended the University of Cambridge at the age of 19 (yes, he was always a pretty smart guy).¹ While there, he was influenced by the writings of a handful of famous scientists and mathematicians, including Galileo and Copernicus.



Sure, Sir Isaac is most acclaimed for his discoveries on gravity, but he uncovered many other scientific wonders in his lifetime. In fact, his work in the realm of physics was so advanced, he was knighted by the King of England... hence the “Sir” preceding his name.² By 1665, our friend, just 22 at the time, began developing a theory that would eventually pave the way to a field in math which some of you may be familiar with. That’s right, calculus -- a pretty fundamental branch of math if I do say so myself, after having basked in its glory for nine months.

Newton’s discoveries in calculus first began as a method of finding the slope at any point on a fluctuating curve, which gave way to differentiation (derivatives), integration (anti-derivatives), and the development of the Fundamental Theorem of Calculus³ -- all that fun stuff we AP Calculus girls have been grappling with all year.

Things got a little sticky for Newton, however, because while he was making strides in his calculus discoveries, a German gentleman by the name of Leibniz was, too -- only Leibniz was publishing his findings.⁴ Thus was born a bitter dispute between the two mathematicians that would later become known as the “Great Sulk.” In fact, the issue grew

so heated, that as the President of the Royal Society, Newton appointed an “impartial” committee to determine whether he or Leibniz was the true inventor of calculus.⁵ Today, humanity has come to the agreement that both mathematical giants discovered it independently of one another.

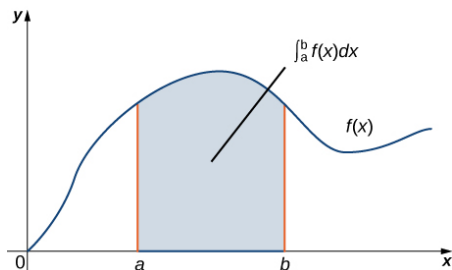


Illustration of the Fundamental Theorem of Calculus, developed by Newton.

Regardless, on behalf of the entire AP Calculus class and Mrs. Feinberg, I would like to express our deepest thanks and appreciation to Sir Isaac Newton for playing such an integral (pun intended) part in the world of mathematics and for making our universe and our classroom a better, more purposeful place.

1 http://mrnussbaum.com/pioneers/isaac_newton/

2 <https://askabiologist.asu.edu/sir-isaac-newton>

3 http://www.storyofmathematics.com/17th_newton.html

4 <http://www.mscs.dal.ca/~kgardner/History.html>

1646- 1716

Gottfried Leibniz

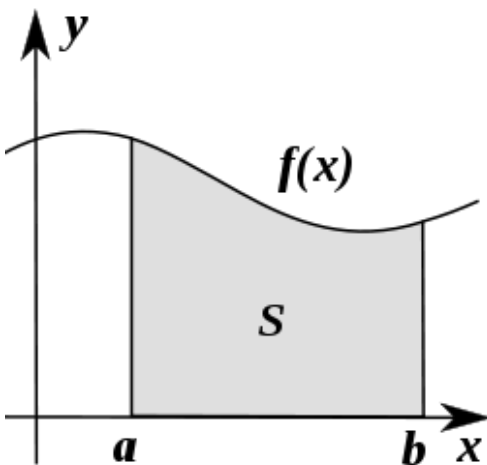
Esther Butler

Gottfried Leibniz was born in Leipzig, Germany in 1646 and passed away in Hanover, Germany in 1716. He was a prodigy, and at age fourteen he attended the University of Leipzig. He received his doctorate from the University of Altdorf in 1666.¹ Leibniz studied in Paris in 1672 under



Christiaan Huygens, a mathematician.⁴ Leibniz discovered the binary system and used it to invent the first calculator, which could add, subtract, multiply, and divide.¹ His greatest mathematical contribution is his discovery of a new mathematical notation, infinitesimal Calculus, which combined differential and integrational calculus.² However, Sir Isaac Newton discovered Calculus, before Leibniz, in reference to

his work on gravity, but Leibniz published his work in 1684, a few years before Newton.¹



Leibniz's notation for infinitesimal calculus is still used today. He assigned "dx" to differential calculus and "∫" to integration.² He gave our Calculus class the ability to separate variables and cross multiply to solve

differentiation and integration equations. Previous mathematicians had worked on differentiation, but Leibniz proved this “algorithmic process” and that it is the inverse of integration. He showed that taking the derivative of the area beneath a curve is the function itself, such that a derivative and integral are inverse operations. With the discovery of differentiation, a tangent to a curve can be found at a specific point, while integration finds the area beneath a given curve.³

1 “Gottfried Leibniz.” *Famous Scientists*. N.p., 2017. Web. 24 May 2017. <<https://www.famousScientists.org/gottfried-leibniz/>>.”Gottfried Leibniz.” Famous

2 “Gottfried Wilhelm Leibniz , the True Inventor of Calculus.” *Famous Mathematician’s Name*. N.p., n.d. Web. 24 May 2017. <<http://www.math.umt.edu/bardsley/courses/181/FamousMathematician/meganbrittany.htm>>.

3 “History of the Differential from the 17th Century.” History of the Differential from the 17th Century. N.p., n.d. Web. 24 May 2017. <<http://www.math.wpi.edu/IQP/BVCalcHist/calc2.html>>.

4 Look, Brandon C. “Gottfried Wilhelm Leibniz.” *Stanford Encyclopedia of Philosophy*. Stanford University, 24 July 2013. Web. 24 May 2017. <<https://plato.stanford.edu/entries/leibniz/>>.

1707-1783

Leonhard Euler

Gabrielle Hawk

Arguably the most published mathematician of all time, Leonhard Euler was never supposed to become a mathematician. Born to a Protestant Minister and the daughter of a minister in Riehen, Switzerland in 1707, clergy was in Euler's blood.

Euler was a dedicated student with a mind for details, especially languages and math. Since his elementary education was not stellar, Euler's father supplemented his schooling with extra math lessons. At the age of 14, Euler enrolled in the University of Basel to obtain a Master's in Philosophy.¹ Euler, however, found math more enticing than divinity studies. Luckily for him, Euler's father was friends with Johann Bernoulli - leading mathematician and professor at the university. Euler was therefore given permission to spend extra hours studying math under the mentorship of Bernoulli.

After graduating the University, Euler was offered a position at St. Petersburg Academy. There, he eventually became the Chair of both Geography and Mathematics.² Euler was often approached with difficult problems and he solved them, calculating complicated problems in his head. For example, Euler solved the Konigsberg Bridge Problem, which became the basis for understanding networks, and the Basel Problem.

In St. Petersburg, Euler married Katherine Gsell and together they had thirteen children. Euler also became a consultant for the Russian government. All the while, Euler was adding new understandings



to nearly every branch of math known at the time. He introduced symbols for both known math terms and ones he generated, such as e , $f(x)$, and i , symbols we used throughout high school math. Euler worked a lot on infinite series and also produced Euler's Identity: $e^{i\pi} = -1$, used to simplify derivations.¹ He also expanded on Newton's Laws of Motion with Euler's Laws of Motion - laws that could be applied to physical objects, unlike Newton's laws reserved for particles.

Euler's published works include *Mechanica*, *Foundations of Differential Calculus*, and *Letters to a German Princess*, in which he explains 200 math and science concepts in layman's terms for a German princess. He also produced groundbreaking work on shipbuilding, music, and acoustics and published *Classical Number Theory* with Christian Goldbach.³

Despite going completely blind at the age of 64, Euler increased his output of math publications.² He produced such volumes of work that when he died in 1783, it took another 48 years for St. Petersburg Academy to finish publishing all that he wrote. It took another century for a complete collection of Euler's work to be published in 1909 as *Omnia Opera*, which includes over 800 papers and books on math and physics.

Twentieth-century mathematician Clifford Truesdell once said that about one third of all math, physics, and engineering mechanics published in the nineteenth century can be attributed to Leonhard Euler.³

"Leonhard Euler." Famous Scientists. N.p., n.d. Web. May 2017.

O'Connor, J. J., and E. F. Robertson. "Leonhard Euler." Euler biography. N.p., 1998. Web. May 2017.

Taylor, Peter. "Leonhard Euler." Australian Mathematics Trust. N.p., Mar. 2001. Web. May 2017.

1777-1855

Carl Friedrich Gauss

Rachel Klamen

Carl Friedrich Gauss was born in Brunswick, Germany on April 30, 1777 to Dorothea Benze and Gebhard Dietrich Gauss.¹ He is known as the greatest German mathematician of the 19th century.⁵ He worked in number theory, physics, astronomy, and geodesy.⁵

Gauss was not only a mathematician. Some of his non-math achievements include: the invention of the telegraph, the invention of the heliotrope, an instrument that uses a mirror to reflect the sunlight to mark positions in a land survey and working in the field of electromagnetism.³ Because of his work in electromagnetism, the unit of magnetic induction is called the gauss.³



Gauss' father didn't want him to go to school, because he wanted Gauss to go into the family trade. His mother however, realized his brilliance and encouraged him to attend school.⁵

Gauss was a child prodigy.³ When he was 10 years old, his teacher told the class to write all the numbers from 1-100 and add them together. The teacher expected everyone to take a long time, but Gauss was able to finish in a few seconds. He said he figured it out quickly because the 50 pairs of numbers (1+100, 2+98...) all =101, and 50 pairs x 101= 5050. We used the basis for his idea to calculate the sum of a finite arithmetic sequence in Pre-Calculus.⁵

When he was 14, Gauss began learning with Carl Wilhelm Ferdinand, Duke of Brunswick, and the Duke supported him throughout his studying in college.⁵ The Duke of Brunswick sponsored him to attend College Carolinum and University of Gottingen.³ While in Carolina

College, Gauss discovered that he was able to draw a 17-sided polygon with a compass and ruler. This finding led Gauss to study math, and uncover many new discoveries.⁵ Besides just finding a way to draw a 17-sided polygon, something mathematicians were working on for years, he also came up with a formula to find all regular polygons that can be drawn with a compass and straightedge.⁴

Gauss formulated the Fundamental Theorem of Algebra⁵, which states that any polynomial that has a degree (highest exponent) of n , has n complex roots, and any polynomial with a degree higher than 1 has at least one complex root.² This theorem is used in pre-calculus, when finding zeroes and graphing polynomial functions.

Gauss also worked with astronomy.³ In 1801 Giuseppe Piazzi, an Italian scientist found what he thought was a comet.¹ He lost the location, but Gauss was able to relocate it and find its exact location⁵, and he said he did the logarithmic calculations in his head.³ (The object was actually the asteroid Ceres.)¹ While working with this calculation, Gauss also found a new method for determining the orbits of asteroids. Through these discoveries he was given a job at Gottingen College, and stayed there until he died.⁵

Before Gauss, people used imaginary numbers to solve equations, but no one really knew how imaginary numbers related to real numbers. Other mathematicians such as Jean-Robert Argand and Dane Caspar Wessel had already expressed complex numbers on graphs, but Gauss made it popular, and made the $a + bi$ notation popular as well.³ Gauss also developed the Gaussian distribution, Gaussian function and the Gaussian error curve, also known as the bell curve, used to determine normal distribution.³

Carl Friedrich Gauss died on February 23, 1855 in Hanover, Germany.⁴ Throughout his life, Gauss kept all his mathematical work

in secret diaries, and didn't want to make any of it public until it was perfect.⁵ In his diaries were over 140 discoveries he made, and it was not found until 40 years after he died.¹ Gauss is known as one of the three greatest mathematicians, along with Archimedes and Newton,⁵ and also known as the "Prince of Mathematicians".³

1 "Carl Friedrich Gauss." Famous Scientists. famousscientists.org 25 Jul. 2016. Web. 6/11/2017 <www.famousscientists.org/carl-friedrich-gauss/>

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3 Mastin, Luke. "19TH CENTURY MATHEMATICS - GAUSS." *The Story of Mathematics* . N.p., n.d. Web. 11 June 2017 www.storyofmathematics.com/19th_gauss.html

4 Siegrist, Kyle. "Carl Friedrich Gauss." *Carl Gauss*. N.p., n.d. Web. 11 June 2017. <<http://www.math.uah.edu/stat/biographies/Gauss.html>>

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1826 - 1866

Bernhard Riemann

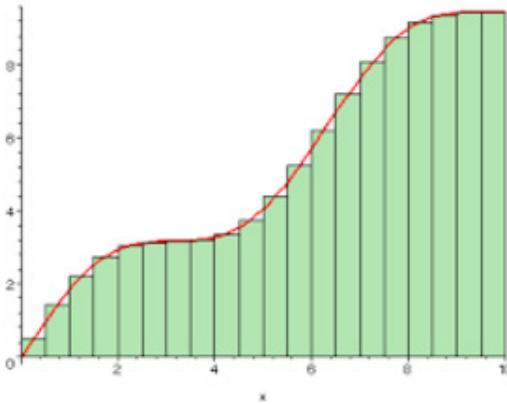
Ayelet Huberfeld

Bernhard Riemann was born on September 17, 1826 in Hanover, Germany. He was a gifted math student as a child, and although he was originally studying theology and philosophy in school, he eventually switched his focus to math.¹ He studied math in the University of Göttingen and later at the University of Berlin. He returned to the University of Göttingen where he completed his doctoral thesis under the guidance of Carl Friedrich Gauss. In 1852, he was given the position of unpaid lecturer, worked his way to being a full professor, and was elected to the Berlin Academy of Sciences in 1859. He got married in 1862, and soon after, contracted tuberculosis. In 1866, Riemann died in Italy, where he had traveled for health reasons.²



Riemann was not recognized for his great achievements while he was alive, since many of his papers were published after his death. We mentioned Riemann in AP Calculus with his contribution to calculating the area under a curve, called the Riemann Sum. Using the Right and Left Riemann Approximation Methods, the integral could be estimated by adding up the area of rectangles drawn under the curve.³ Another one of Riemann's main contributions was in the study of graphing complex equations on what he called a "Riemann Surface." He argued that geometry need not be bound by 2 or 3 dimensional space. Rather, it can extend to even potentially infinite dimensions. This discovery was integral in Albert Einstein's theory of relativity.⁴ Riemann's contributions to Calculus were very significant, and everyone, not just

math students, owes him a debt of gratitude.



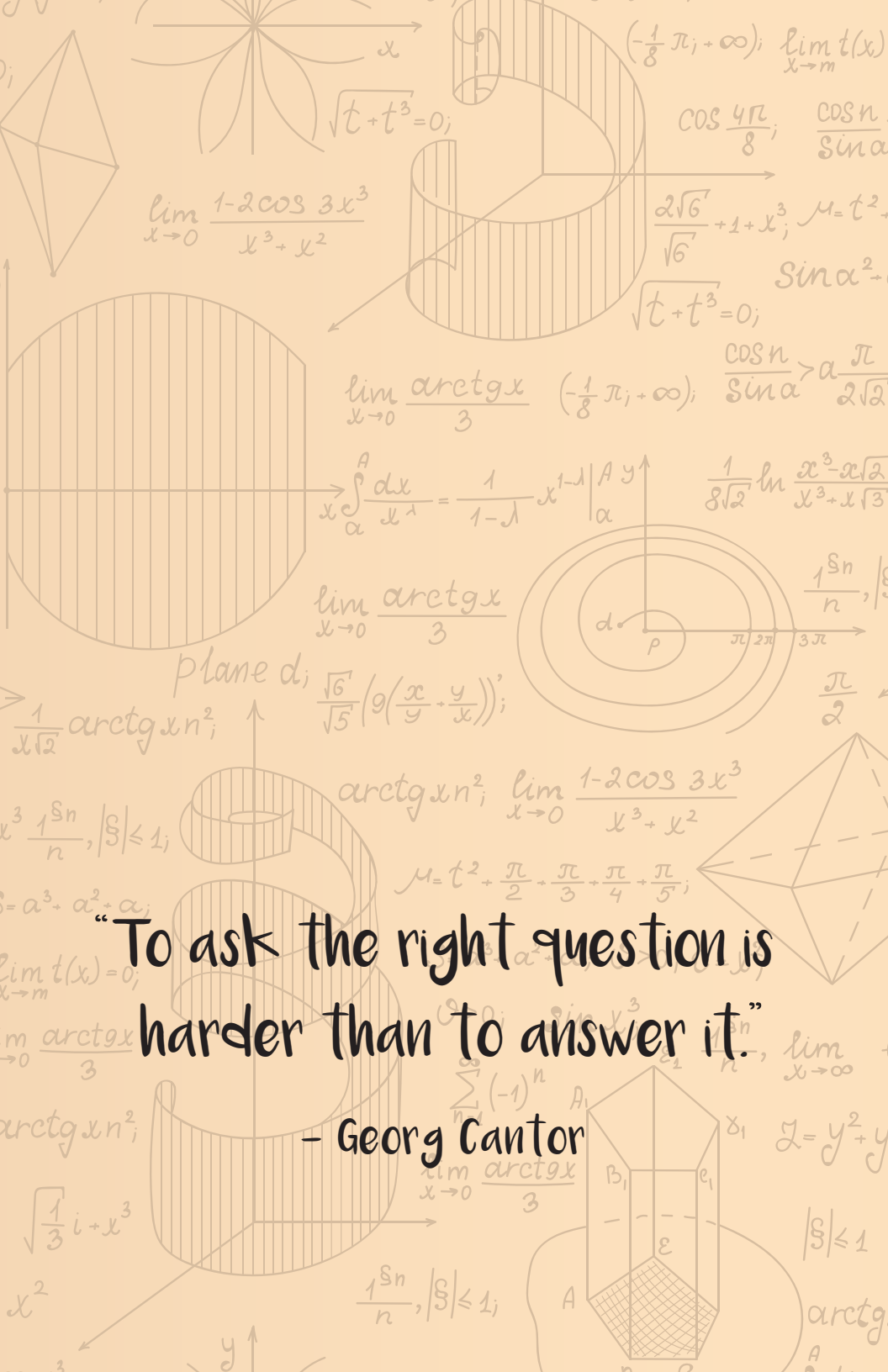
An illustration of Riemann's method for calculating the area under a curve.

1 Middlebury Blog Network, "Bernhard Riemann Biography." Middlebury Blog Network. 2 October 2011. Web. <http://sites.middlebury.edu/fyse1229hunsicker/biography/>

2 Hoffman, Mike. "Bernhard Riemann" U. S. Naval Academy. 6 June 2017. Web. <https://www.usna.edu/Users/math/meh/riemann.html>

3 Middlebury Blog Network, "Reimann and Calculus" Middlebury Blog Network. 2 October 2011. Web. <http://sites.middlebury.edu/fyse1229hunsicker/riemann-and-calculus/>

4 Gray, Jeremy John. "Bernhard Riemann" Encyclopaedia Britannica. 29 March 2017. Web. <https://www.britannica.com/biography/Bernhard-Riemann>



$$\lim_{x \rightarrow 0} \frac{1 - 2 \cos 3x^3}{x^3 + x^2}$$

$$\lim_{x \rightarrow 0} \frac{\arctg x}{3}$$

$$\int_{\alpha}^A \frac{dx}{x^{\lambda}} = \frac{1}{1-\lambda} x^{1-\lambda} \Big|_{\alpha}^A$$

$$\lim_{x \rightarrow 0} \frac{\arctg x}{3}$$

$$\text{plane } d; \sqrt{\frac{1}{5}} \left(9 \left(\frac{x}{y} + \frac{y}{x} \right) \right)^2$$

$$\frac{1}{x\sqrt{2}} \arctg x n^2$$

$$\frac{1}{n} \arctg x n^2, |S| \leq 1$$

$$a^3 - a^2 + a$$

$$\lim_{x \rightarrow m} t(x) = 0$$

$$\lim_{x \rightarrow 0} \frac{\arctg x}{3}$$

$$\arctg x n^2$$

$$\sqrt{\frac{1}{3} i + x^3}$$

$$x^2$$

$$\frac{1}{n} \arctg x n^2, |S| \leq 1$$

- Georg Cantor

$$\lim_{x \rightarrow 0} \frac{\arctg x}{3}$$

$$Z = y^2 + y$$

$$|S| \leq 1$$

$$\arctg$$

$$\left(-\frac{1}{8} \pi; +\infty\right); \lim_{x \rightarrow m} t(x)$$

$$\cos \frac{4\pi}{8}; \frac{\cos n}{\sin a}$$

$$\frac{2\sqrt{6} + 1 + x^3}{\sqrt{6}}; \mu = t^2$$

$$\sin a^2$$

$$\sqrt{t+t^3}=0$$

$$\lim_{x \rightarrow 0} \frac{\arctg x}{3} \left(-\frac{1}{8} \pi; +\infty\right); \frac{\cos n}{\sin a} > a \frac{\pi}{2\sqrt{2}}$$

$$\frac{1}{8\sqrt{2}} \ln \frac{x^3 - x\sqrt{2}}{x^3 + x\sqrt{2}}$$

$$\frac{1}{n} \arctg x n^2$$

$$\frac{\pi}{2}$$

$$\arctg x n^2; \lim_{x \rightarrow 0} \frac{1 - 2 \cos 3x^3}{x^3 + x^2}$$

$$\mu = t^2 + \frac{\pi}{2} + \frac{\pi}{3} + \frac{\pi}{4} + \frac{\pi}{5}$$

$$\sum_{n=1}^{\infty} (-1)^n A_n; \lim_{x \rightarrow \infty} \frac{1}{n} \arctg x n^2$$

$$\lim_{x \rightarrow 0} \frac{\arctg x}{3}$$

