PROVE IT!

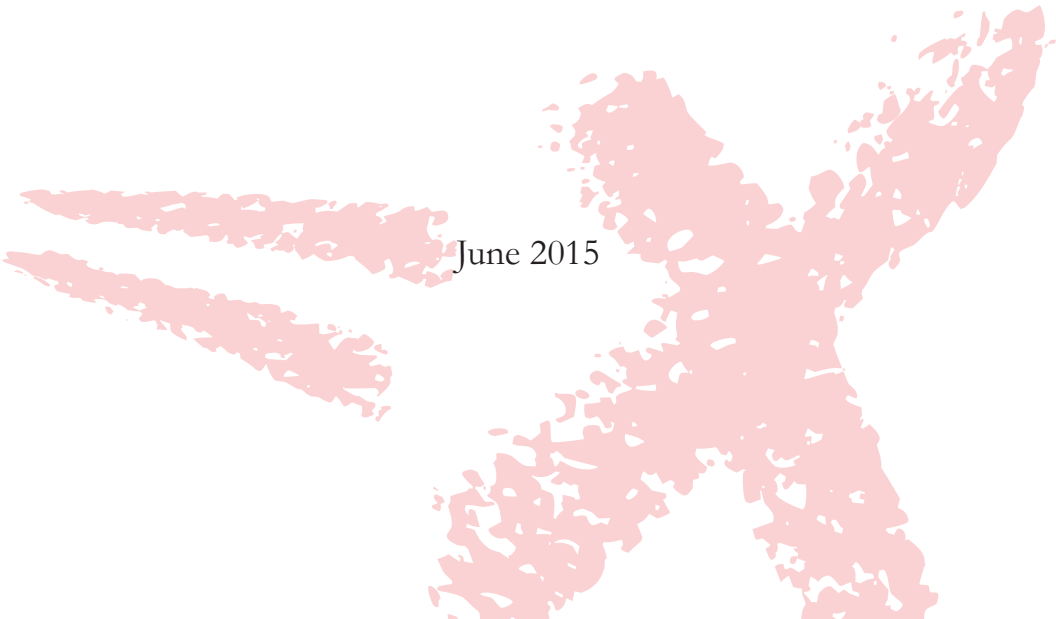
Math Riddles Demystified

Produced by the AP Calculus Class
of
Manhattan High School for Girls

June 2015

Prove It!

A collection of Math Challenges
produced by the Math Department
of
Manhattan High School for Girls



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"Pure mathematics," posited Albert Einstein, "is in its way, the poetry of logical ideas."

At MHS, our goal for mathematics education is to develop our students' critical thinking and quantitative and problem solving skills while giving them a thorough understanding of algebra, geometry, and trigonometry calculus. These essential foundations will enable our students to continue their education in scientific and technical fields such as accounting, business, actuarial science, architecture, engineering and pharmacology.

But, perhaps the greatest benefit of these essential mathematical foundations is that our girls are able to now access clearer and more organized minds as they approach diverse challenges and contexts.

Kudos to our students who appreciate and desire robust learning and to Mrs. Goldie Feinberg, our esteemed Math Chair!

Ms. Estee Friedman
Principal
General Studies

Foreword

Mrs. Goldie Feinberg, Math Chair

At Manhattan High School we encourage our students to understand and prove mathematical facts and formulae; not just blindly memorize a series of steps. As a result, our students aren't satisfied until they grasp the depth and nuances of the material. In *Prove It!* each student took a math riddle and set out to demystify it and demonstrated that when the results are examined, they are logical, not magical. The analytical skills displayed by these AP Calculus students are impressive. It is my hope that as perennial learners, they will always seek to analyze and question.

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Only Nine Remain

Shifra Abittan

Math Trick	Example
Step 1: Choose a four digit positive integer that contains at least two different digits.	1234
Step 2: Rearrange the four digit number.	4321
Step 3: Subtract the smaller four digit number from the larger.	4321 - 1234 = 3087
Step 4: Circle one digit. You may not circle zero.	3087
Step 5: Re-write the number without the circled digit.	308
Step 6: Compute the sum of the digits.	3 + 0 + 8 = 11
Step 7: Write down the next multiple of nine that is larger than the sum from Step 6.	18
Step 8: Subtract the sum of the digits (Step 6) from the multiple (Step 7).	18 - 11 = 7
Step 9: The difference should equal the circled number.	7 = 7

The Way it Works...

The rule of divisibility by 9 says that a number is divisible by 9 if the sum of its digits is divisible by 9. The sum of a number's digits divided by 9 will always have the same remainder as the original number divided by 9.

Therefore, the numbers in Step 1 and Step 2 when divided by 9 will yield the same remainder.

Subtracting Step 1 and Step 2 subtracts the remainder away, leaving you with a number divisible by 9. Next, you remove a digit; name it D. The sum of the remaining digits is D less than a multiple of 9. Let us call this sum S. The next multiple of 9 after S is S +D. When you subtract S + D - S, the both variables S cancel and the result is D, the original number that you circled.

Puzzling Predictions

Talia Alper

Math Trick

Step 1: Pick a number.

Step 2: Multiply by 2.

Step 3: Add 16.

Step 4: Multiply by a half.

Step 5: Subtract your original number.

Is your answer 8?

Want to try again?

Step 1: Pick another number.

Step 2: Multiply by 2.

Step 3: Add 84.

Step 4: Multiply by a half.

Step 5: Subtract your original number.

Is your answer 42?

The Way it Works...

1. Pick a number.	x
2. Multiply by 2.	2x
3. Add 16.	2x + 16
4. Multiply by a half.	(2x + 16) ÷ 2
5. Subtract your original number.	[(2x + 16) ÷ 2] - x Simplify: (2x) ÷ 2 + 16 ÷ 2 -x x + 8 - x
Answer is 8.	8

Try it again!

1. Pick another number.	x
2. Multiply by 2.	2x
3. Add 84.	2x + 84
4. Multiply by a half.	(2x + 84) ÷ 2
5. Subtract your original number.	[(2x + 84) ÷ 2] - x Simplify: (2x) ÷ 2 + 84 ÷ 2 - x x + 42 - x
Answer is 42.	42

You can try it with any number!
The final answer will ALWAYS be the number
that was added, divided by 2.

All Roads Lead to 1089

Nechama Dembitzer

Math Trick	**Example**
Step 1: Think of a three digit number.	462
Step 2: Arrange the three digit number into descending order.	642
Step 3: Arrange the original number in ascending order and subtract it from the number in Step 2.	642 - 246 = 396
Step 4: Remember the answer from Step 3 and reverse the order of its digits.	693
Step 5: Add those two numbers.	396 + 693

Did you get 1089?

The Way it Works...

Pick a three digit number, ABC.

C < B < A
Since you are subtracting larger digits from smaller digits, you must carry between columns.

Hundreds	Tens	Ones
A - 1	B + 9	C + 10
- C	B	A
A - 1 - C	9	C + 10 - A

Hundreds	Tens	Ones
A - 1 - C	9	C + 10 - A
+ 10 + C - A	9	A - 1- C
9	18	9

900 + 180 + 9

1089

Age is But a Number

Avigail Dreifus

Math Trick	Example
Step 1: Pick a number between 1 and 10.	4
Step 2: Multiply by 2.	8
Step 3: Add 5.	13
Step 4: Multiply by 50.	650
Step 5: If you already had your birthday this year, add 1765. — OR — If you have not had your birthday this year, add 1764.	2414
Step 6: Subtract from the sum the year you were born. (Example: 1997)	417
Step 7: You are left with a three digit number. The first of the three numbers is the original number you picked in Step 1, and the last two digits are your age.	4 is the original number, and 17 is my age

The Way it Works...

Let x = a number between 1 and 10

Step 1: x

Step 2: 2x

Step 3: 2x + 5

Step 4: 50 (2x + 5) = 100x + 250

Step 5: 100x + 2015

— OR—

100x + 2014

Step 6: 100x + 17

Step 7: The original number, x, has been multiplied by 100, so is now in the hundreds place, and the last two digits are your age

Guess 2 Numbers

Ahuva Forman

Math Trick	Example
Step 1: Pick a number from 0 to 9.	2
Step 2: Double the number.	4
Step 3: Add 5.	9
Step 4: Multiply by 5.	45
Step 5: Pick another number from 0 to 9.	4
Step 6: Add this to the answer from Step 4.	49
Step 7: Subtract 25.	24
Step 8: The first digit of the answer after subtracting 25 is the first number you picked. The second digit is the second number you picked.	First Digit -2 Second Digit -4

The Way it Works...

n

2n

2n + 5

5(2n + 5) □ 10n + 25

m

10n + 25 + m

10n + 25 + m - 25 = 10n + m

Multiplying the first digit, n, by 10, places the first digit in the tens place. Adding the second digit, m, places it in the ones column.

Family Figures

Miri Fried

Math Trick

Step 1: Multiply the number of your living brothers by 2.

Step 2: Add 3.

Step 3: Multiply by 5.

Step 4: Add the number of your living sisters.

Step 5: Multiply by 10.

Step 6: Add the number of your living grandparents.

Step 7: Subtract 150.

The first digit of your new result is the number of living brothers you have. The second digit is the number of living sisters you have. The last digit is the number of living grandparents you have.

The Way it Works...

1. Multiply the number of your living brothers by 2.	2a
2. Add 3.	2a + 3
3. Multiply by 5.	10a + 15
4. Add the number of your living sisters.	10a + 15 + b
5. Multiply by 10.	100a + 150 + 10b
6. Add the number of your living grandparents.	100a + 150 + 10b + C
7. Subtract 150.	100a + 10b + C

100A= Living Brothers
10B= Living Sisters
C= Living Grandparents

Caller ID

Yaffa Jacobson

Math Trick

Step 1: Grab a calculator.

Step 2: Type in the first three digits of your phone number without the area code.

Step 3: Multiply by 80.

Step 4: Add 1.

Step 5: Multiply by 250.

Step 6: Add the last four digits of your phone number.

Step 7: Add the last four digits of your phone number again.

Step 8: Subtract 250.

Step 9: Divide number by 2.

Did you get your phone number?

The Way it Works...

1. abc

2. 80abc

3. 80abc + 1

4. 20000abc + 250

5. 20000abc + 250 + defg

6. 20000abc + 250 + 2defg

7. 20000abc + 2defg

8. 1000abc + defg

By multiplying abc by 1000, you put abc in the thousands place.

THREE Times the Charm

Yakira Klein

Math Trick

Step 1: Choose any positive integer.

Step 2: Square it.

Step 3: Add the result to the original number.

Step 4: Divide by your original number.

Step 5: Add 17.

Step 6: Subtract your original number.

Step 7: Divide by 6.

Did you get 3?

The Way it Works...

1. x

2. x^2

3. $x^2 + x$

4. $x + 1$

5. $x + 18$

6. 18

7. $18 \div 6 = 3$

Fair and Square

Aliza Lobell

Math Trick	Example
Step 1: Pick a two digit number ending in 5.	35
Step 2: Take the "tens" digit of the number and add 1.	3 + 1 = 4
Step 3: Multiply the original "tens" digit of the number you chose by the answer from Step 2.	4 • 3 = 12
Step 4: Put 25 after the number found in Step 3.	1225

The Way it Works...

1. Let the two digit number equal $10a + 5$.

2. Add 1 to the tens digit, $a + 1$.

3. Multiply the new tens digit, $(a + 1)$, by the original tens digit, a.
   $a(a + 1) = (a^2 + a)$.

4. Multiply the number from Step 3 by 100 to put it into the hundreds place. Add 25.
   $100(a^2 + a) + 25 = 100a^2 + 100a + 25$

5. To prove they are the same, square the two digit number.
   $(10a + 5)^2 = 100a^2 + 100a + 25$

Order of Operations

Penny Rabin

Math Trick

Step 1: Chose a number from 1 to 6.

Step 2: Multiply by 9.

Step 3: Multiply by 111.

Step 4: Multiply by 1001.

Step 5: Divide by 7.

The answer will have only the digits 124578 but not necessarily in that order.

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The Way it Works...

1. a

2. 9a

3. 999a

4. 999,999a

5. 999,999a / 7 = 142857a.

That has only those numbers and only all once.

142857 • 1 = 142857

142857 • 2 = 285714

142857 • 3 = 428571

142857 • 4 = 571428

142857 • 5 = 714285

142857 • 6 = 857142

Tri - Math

Esther Rothman

Math Trick

Step 1: Think of any three digit number where each digit is the same. (ex: 222, 444)

Step 2: Add up the digits.

Step 3: Divide the three digit number you chose in Step 1 by the sum of the digits you found in Step 2.

Did you get 37?

The Way it Works...

1. A three digit number each digit being the same number:
   $100x + 10x + x = 111x$

2. Add the digits:
   $x + x + x = 3x$

3. $\frac{111x}{3x} = 37$

Not Everything is Black & White

Tova Schwartz

Math Trick

Two terrorists kidnapped 11 people and held them hostage in a hat shop. Can you help the hostages outwit the terrorists?

The terrorists will line up the 11 men in a single file row all facing the person/people in front of them, they will then place either a black or white hat on each of the hostages' heads. The terrorist will start at the back of the line and ask each hostage what color hat he is wearing. The hostage *must* answer black or white. If he guesses correctly he is freed, and if not, he is sentenced to jail for life in Guatemala.

There is one catch:

The hostages are allowed to discuss a plan of action before they are forced to line up and have a hat placed on their head. While they are planning, a computer programmer who was among them came up with a code that could save them --- what was it?

The Way it Works...

The hostages devise a plan as follows:

The last person in line, Chuck, the first person the terrorists questions, will be *moser nefesh* and take a 50/50 chance at being freed versus being stuck in Guatemala for life in order to save the other ten people.

The hostages set up a code. Chuck will look in front of him and count the number of white hats. If there are an even number of white hats he will say "White" and if there are an odd number of white hats he will say "Black."

How does this help?
Let's say this is the order of the hats in line:

Black
White
Black
Black
Black
White
Black
White
White
Black
White - This is Chuck.

Chuck looks ahead of him and sees an even number of white hats, four. He says white. (He happens to be lucky in this scenario and is wearing a white hat and will be spared life in prison.) When he says "white" everyone knows that there is an even number of white hats.

Then, person ten looks in front of him and sees four white hats. He knows that there is an even number of white hats; therefore, he can't be wearing a white hat. If he was wearing a black hat, Chuck would have seen an odd number of white hats and said, "Black."

Person nine sees three white hats in front of him and remembers that the person behind him said "Black." Thus, he knows in order for Chuck to have seen an even number of white hats he must have a white hat etc.

Race to the Finish

Mrs. Goldie Feinberg

Math Trick	Example
Step 1: Pick two numbers from 1 – 9.	4 and 9
Step 2: Add the two numbers together.	4 + 9 = 13
Step 3: Add the second and third numbers to generate a fourth number.	9 + 13 = 22
Step 4: Add the third and fourth to generate a fifth number.	13 + 22 = 35
Step 5: Keep repeating the process until you have ten numbers in a list.	4, 9, 13, 22, 35, 57, 92, 149, 241, 390
Step 6: Ask someone to use a calculator to add all ten numbers together, while you calculate the number in your head faster than the calculator!	Answer: 1012

The Way it Works...

Pick any two digits, a and b.

a

b

a + b (sum of the first and second digits)

a + 2b (sum of second and third digits)

2a + 3b

3a + 5b

5a + 8b ⟶ Seventh number

8a + 13b

13a + 21b

21a + 34b

Add them all together… You get 55a + 88b which is eleven times the seventh number, 5a + 8b.

[To multiply by 11 quickly in your head you need to know the 11 times tables trick.]

You should be able to get to the answer faster than the calculator every time!

"I think math is a hugely creative field, because there are some very well-defined operations that you have to work within.

You are, in a sense, straightjacketed by the rules of the mathematics. But within that constrained environment, it's up to you what you do with the symbols."

- Brian Greene, Physicist

Acclaimed for his Groundbreaking Discoveries in his field of Superstring Theory